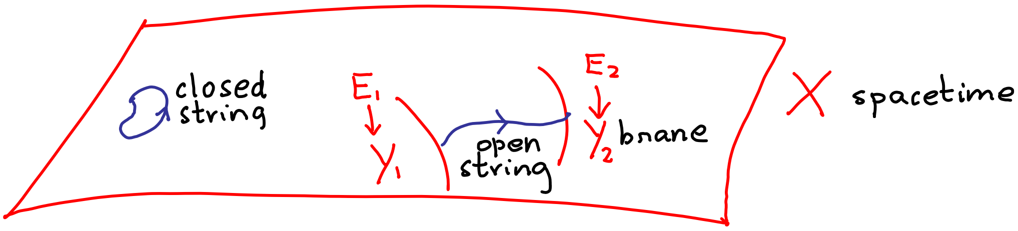
Costello. String theory for mathematicians

2017/5 Perimeter

Topological strings : A and B models.



§ B-model.

CY manifold $X^{n_{\mathbb{C}}}$ (odd) $\supset Y^{k_{\mathbb{C}}}$ cpx. submfd

eg. $\mathbb{C}^n \supseteq \mathbb{C}^k$ B-Branes

$\{\text{Open string states}\} = \Omega^{0,*}(\mathbb{C}^k)[\varepsilon_1, \cdots, \varepsilon_{d-k}] \otimes \mathfrak{gl}_N$

↳ Connecting $\mathbb{C}^k$ with itself; N branes wrapping $\mathbb{C}^k$

dga : $\bar\partial$, $\wedge$. $\{\varepsilon_i, d\bar{z}_j\} = 0$

trace : $\mathcal{A} \triangleq \Omega^{0,*}(\mathbb{C}^k)[\varepsilon_i] \otimes \mathfrak{gl}_N \xrightarrow{\mathrm{Tr}} \mathbb{C}$

$$\mathrm{Tr}(A) = \int_{\mathbb{C}^{k|d-k}} dz_1 \cdots dz_k \, d\varepsilon_1 \cdots d\varepsilon_{d-k} \, \mathrm{Tr}_{\mathbb{C}^N} A$$

Open String Field Theory

Action $S : \Pi\mathcal{A} = \{\text{fields}\} \longrightarrow \mathbb{C}$

$$S(\alpha) \triangleq \mathrm{Tr}\left(\tfrac{1}{2}\alpha\bar\partial\alpha + \tfrac{1}{3}\alpha^3\right)$$

Eg. ε_j's are bosonic scalar in open string field theory.

Relation to physical String.

type IIB string theory on $\mathbb{R}^{10}$,

SUSY of type IIB are $S_+ \oplus S_+$ (dim=16)

(Topo. twist require $Spin(10) \longrightarrow SO(2,\mathbb{R})$ perturbatively

$\cup$ non-

$SO(2,\mathbb{Z})$ perturbatively

$\nexists$ such Q.)

Can twist by $SU(5)$-inv. Q.

($\exists$ 2 supercharges Q_1, Q_2 inv. under $SU(5) \subseteq Spin(10)$

($G_R = SO(2)$ can rotate Q_1 to Q_2))

- Brane is preserved by Q

$\longleftrightarrow$ complex subspace $\mathbb{C}^k \subset \mathbb{C}^5 = \mathbb{R}^{10}$

- Twisting. $Q_{BRST} \longmapsto Q_{BRST} + Q$

Closed String fields

Add Q to BRST operator closed string states.

$\dot{Q}$ $\dot{Q}$ $\dot{Q}$ ← R-R states given by Q

($Q \in$ local SUSYs which give ghosts in type IIB SUGRA
In type IIB, super-ghost has expectation value given by Q.) (?)

<u>Conjecture</u>: This $SU(5)$-inv. twist of type IIB on $\mathbb{R}^{10}$
$\equiv$ topo. B-model on $\mathbb{C}^5$.

Physical IIB		Topological
D_{2k-1}-brane on $\mathbb{C}^k \subseteq \mathbb{R}^{10} = \mathbb{C}^5$	$\Rightarrow$	B-brane on $\mathbb{C}^k \subseteq \mathbb{C}^5$
Open-string states, $Q_{BRST} + Q$	$\overset{\sim}{\Rightarrow}$	(open string states, $\bar{\partial}$)
holom. twist of low energy theory on the brane	$\Rightarrow$	Open-string field theory we discussed. $\Pi\Omega^{0,*}(\mathbb{C}^k)[\varepsilon_i] \otimes \mathfrak{gl}_N$
⋮		⋮

Theorem (L. Baulieu)
$SU(5)$-inv. twist of max. SUSY gauge theory on $\mathbb{R}^{10}$.
$\equiv$ hol. CS on $\mathbb{C}^5$.

(holomorphic CS on $\mathbb{C}^{k|5-k}$: field $\Pi\Omega^{0,\bullet}(\mathbb{C}^k)[\varepsilon_1, \dots, \varepsilon_{5-k}] \otimes \mathfrak{gl}_N$

$$S(\alpha) = \int_{\mathbb{C}^{k|5-k}} d\,\mathrm{vol} \left(\frac{1}{2}\mathrm{Tr}(\alpha \bar{\partial} \alpha) + \frac{1}{3}\mathrm{Tr}\,\alpha^3 \right)$$
)

$\overset{\text{Thm}}{\Longrightarrow}$ D9 brane (physical) $\longleftrightarrow$ D9 brane (topo.)

Dim. reduction $\Longrightarrow$ other branes ✓

§ A-model

Topological A-model

(X,ω) Sympl. manifold. Branes $L \subset X$ Lagrangian.

$\{\text{Open-string states}\} = HF^*(L_1, L_2) \sim H^{\frac{\infty}{2}}(\mathcal{L}_{L_1 \to L_2} X)$

$\text{"="}\ \Omega^*(L_1 \cap L_2)(\oplus \text{ string corrections})$. (for us)

Open-string states are $\Omega^*(\mathbb{R}^n)$.

Open-string fields (on N branes) $\Pi\Omega^*(\mathbb{R}^n) \otimes \mathfrak{gl}_N$

Action is $\int_{\mathbb{R}^n} \frac{1}{2} \mathrm{Tr}(\alpha\, d\alpha) + \frac{1}{3} \mathrm{Tr}\, \alpha^3$.

SU(4)-inv twist of type IIA on $\mathbb{R}^{10}$ $\overset{\text{Claim}}{\equiv}$ top. string on $\underbrace{\mathbb{R}^2}_{\text{A-model}} \times \underbrace{\mathbb{C}^4}_{\text{B-model}}$ (?).

D_{2k}-branes $\longleftrightarrow$ $\mathbb{R} \times \mathbb{C}^k$ (with $\mathbb{C}^k \subset \mathbb{C}^4$)

Twist of open string field theory $\longleftrightarrow$ Field theory on brane.

Mixed A-B model

$X \times Y \supset L \times Z$ brane

X: sympl. (A), Y: cpx. CY (B), L: Lagr., Z: holom. submfd

Open-string states (w/o string correction) are

$\Omega^*(L) \otimes \Omega^{0,*}(Z, \Lambda^* N_{Y/Z})$ ← normal bundle to Z in Y.

Eg. $\mathbb{R}^{2k}_A \times \mathbb{C}^{5-k}_B$
$\cup$
$\mathbb{R}^k \times \mathbb{C}^\ell$

(k odd ⇒ twist of IIA.
(k even ⇒ twist of IIB.

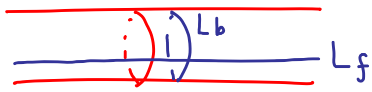
open string states are

$C^\infty(\mathbb{R}^k \times \mathbb{C}^\ell)[dx_1, \dots, dx_k, d\bar{z}_1, \dots, d\bar{z}_\ell, \varepsilon_1, \dots, \varepsilon_{5-k-\ell}]$

$d = d_{\mathbb{R}^k} + \bar{\partial}_{\mathbb{C}^\ell}$

T-duality.

A-model on a cylinder $= T^*S^1$



Naively, OS states for L_f are $\Omega^*(L_f) = \Omega^*(\mathbb{R})$.

Including α'-corrections, $HF^*(L_f, L_f) = \bigoplus_{n \in \mathbb{Z}} \mathbb{C} \cdot n$ ← wrap S^1

Other brane, Naive picture is correct,

$HF^*(L_b, L_b) = \Omega^*(L_b) \simeq \Omega^*(S^1)$.

Also $\mathrm{Fuk}_{wr}(T^*M) \simeq \mathrm{Rep}(C_*(\Omega_x M))$ ← singular chains, based loop space

(eg. $\Omega_x S^1 \simeq \mathbb{Z}$, $C_* \Omega_x S^1 \simeq \mathbb{C}[z^\pm]$)

Fiber at x ⇒ $C_* \Omega_x M$, regular rep.
Base ⇒ $\mathbb{C}$, augmentation rep.

Mirror of this is B-model on $\mathbb{C}^\times$

2 basic branes $\mathcal{O}_{\mathbb{C}^\times} \longleftrightarrow L_f$ (T-dual)
$\mathcal{O}_{pt} \longleftrightarrow L_b$

Open-string states

$\mathcal{O}_{\mathbb{C}^\times} \rightsquigarrow \Omega^{0,*}(\mathbb{C}^\times) \simeq \mathbb{C}[z^\pm]$
$\mathcal{O}_{pt} \rightsquigarrow \mathbb{C}[\varepsilon] = H^*(S^1)$

these match.

Lift this to 10 d

IIA can be twisted to give $(\mathbb{R}\times S^1)_A \times \mathbb{C}^4_B$,

IIB $\mathbb{C}^\times_B \times \mathbb{C}^4_B$,

If $\mathbb{C}^k\subset\mathbb{C}^4$

$\mathbb{R}\times pt\times\mathbb{C}^k \rightsquigarrow \mathbb{C}^\times\times\mathbb{C}^k$: D_{2k} brane $\rightsquigarrow$ D_{2k+1} brane

$pt\times S^1\times\mathbb{C}^k \rightsquigarrow pt\times pt\times\mathbb{C}^k$: D_{2k} brane $\rightsquigarrow$ D_{2k-1} brane

Closed String Sector

A-model.

X symplectic manifold

Closed string states are $\Omega^*(X)$ (first approx., field theory limit)

(More fancy: $SH^\cdot(X)$ Sympl. cohom.)

$X=T^*M$, $SH(X)=H.(LM)$

B-model

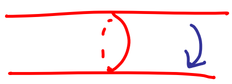
Y complex manifold

Closed string states are $\Omega^{0,*}(Y,\Lambda^\cdot TY)$

$Y=\mathbb{C}^k$

$C^\infty(\mathbb{C}^k)[d\bar{z}_i,\partial_j]$.

S^1-action.



- Physics: Small rotation acts in a Q-exact way on states of TFT. $\partial_\theta=[Q,Q_\theta]$

$\int_{\theta=0}^{\theta=2\pi} Q_\theta$ is an odd Q-closed operator on space of states.

Want states invariant under this.

- A-model. $X=T^*M$, $SH^\cdot(X)=H.(LM)$

$S^1\times LM\longrightarrow LM \rightsquigarrow H.(S^1)\otimes H.(LM)\longrightarrow H.(LM)$

$[S^1]\in H_1(S^1)$ gives a map $H_k(LM)\longrightarrow H_{k+1}(LM)$ which is the above operator.

- B-model The odd operator on $\Omega^{0,*}(Y,\Lambda^\cdot TY)$ is in coordinates z_i, $\Delta=\sum\frac{\partial}{\partial z_i}\frac{\partial}{\partial\partial_i}$.

"Divergence", $\Delta^2=0$.

closed string states are

A-model: $X = T^*S^1$, $H_\bullet(LS^1)$, $LS^1 = \mathbb{Z} \times S^1$, $H_\bullet(LS^1) = \mathbb{C}[z^\pm][\varepsilon]$, ε deg 1 parameter

B-model: $Y = \mathbb{C}^\times$, $\Omega^{0,\bullet}(\mathbb{C}^\times, \wedge^\bullet \mathbb{C}) \simeq \mathbb{C}[z^\pm][\partial]$, $\partial \sim \partial_z$ odd parameter

Δ measure failure of $f z \partial_z$ to preserve $\frac{dz}{z}$.

$\Delta(f z \partial_z) = z f'$

$$\begin{array}{ccc} z^n & \longrightarrow & z^n (z\partial) \\ \int Q_\theta \downarrow & & \downarrow \int Q_\theta \\ z^n \varepsilon & \longrightarrow & z^n \end{array}$$

Closed string states

IIA : $\Omega^\bullet(\mathbb{R}^2) \otimes PV^{\bullet,\bullet}(\mathbb{C}^4) \cap \operatorname{Ker} \partial$

$PV^{\bullet,\bullet}(\mathbb{C}^4)$: $C^\infty(\mathbb{C}^4)[d\bar{z}_i, \partial_{z_i}]$, $\bar{\partial}$ (odd variables)

$\partial = \sum \frac{d}{dz_i} \frac{d}{d\partial_{z_i}}$

IIB : $PV(\mathbb{C}^5) \cap \operatorname{Ker} \partial$

Mostly consider $\bar{\partial}$-cohomology $\operatorname{Ker}\partial \subseteq \mathbb{C}[z_i, \partial_{z_i}]$ (z_i even, ∂_{z_i} odd)

SUSY Type IIB, $\exists$ 32 SUSY $= S_+ \oplus S_+$

{closed string states} $\leftarrow$ Q-cohom.(SUSY alg)

Claim: Q-cohom.(SUSY alg) $\subseteq PV(\mathbb{C}^5)$

$\{\partial_{z_i}, z_j\} = \delta_{ij}$ Schouten bracket

$\mathbb{R}^{10} \otimes_{\mathbb{R}} \mathbb{C} = V \oplus V^*$, V repr. of $SL(5,\mathbb{C})$

$$\begin{array}{ccc} Spin(10,\mathbb{C}) & \curvearrowright & S_+ \\ \text{\rotatebox{90}{$\subseteq$}} & & \| \\ SL(5,\mathbb{C}) & \curvearrowright & \mathbb{C} + \Lambda^2 V + \overbrace{\Lambda^4 V}^{V^*} \end{array}$$

32 spinors of IIB are

$$\begin{array}{cccc} Q \in & \mathbb{C} & \Lambda^2 V & \Lambda^4 V \\ & \mathbb{C} & \Lambda^2 V & \Lambda^4 V \end{array}$$

vector rep. is $\mathbb{R}^{10} \otimes \mathbb{C} = V \oplus V^* = V \oplus \Lambda^4 V$.

Cohomology of $so(10) \xrightarrow{Q} S_+ \oplus S_+ \xrightarrow{[Q,-]} \mathbb{C}^{10}$ is what will survive twisting.

Ker $[Q,-]$ $\quad \Lambda^4 V \xrightarrow[\sim]{[Q,-]} V^*$ $\quad$ ~~$\mathbb{C}$~~ $\curvearrowright$ ~~$\Lambda^2 V$~~ (image of Q under rotation)

$\underline{\mathbb{C} \quad \Lambda^2 V \quad \Lambda^4 V}$, these survive

need $\Lambda^2 V \subset PV(\mathbb{C}^5)$ $\quad V \subset PV(\mathbb{C}^5)$

Answer. $\Lambda^2 V \longrightarrow$ bivectors $\partial_{z_i} \wedge \partial_{z_j}$

$V^* = \Lambda^4 V \longrightarrow$ linear functions z_i

$$\{\partial_{z_i} \wedge \partial_{z_j}, z_k\} = \partial_{z_i} \delta_{jk} - \partial_{z_j} \delta_{ik}$$

exactly relations in SUSY algebra.

Remark: Other twists of IIB include

i) make $\mathbb{C}^5$ non-comm. in some directions,

ii) turn on linear superpotential.

§ D_{2k-1} brane on $\mathbb{C}^k \subseteq \mathbb{C}^5$.

$\rightsquigarrow$ $\frac{1}{2}$-BPS object in physical IIB.

(i.e. Preserve 16 supercharges.)

Can we see Q-cohom. of these 16 supercharges in the twisted theory?

$\underset{w_i}{\mathbb{C}^k} \subseteq \mathbb{C}^5$ (w_i, z_j): coord.

The brane is preserved by z_j $(\sim V^*)$ and $\partial_{w_i} \wedge \partial_{z_j}$ $(\sim \Lambda^2 V)$, but not by others.

$\{z_i, \partial_{w_j} \wedge \partial_{z_k}\} = -\partial_{w_j} \delta_{ik}$ ← translation on the brane

$\exists$ $(5-k) + k(5-k)$ SUSY preserving the brane.

§ <u>D3 brane.</u>

$\mathcal{N}=4$ SUSY 16 supercharges in

$$S_+^{4d} \otimes (\mathbb{C} + W) + S_-^{4d} \otimes (\mathbb{C} + W^*)$$

$$Q = \psi \otimes 1 \in S_+^{4d} \otimes \mathbb{C} \qquad \dim W = 3$$

Q-cohom: $\mathrm{Ker}[Q,-] = S_+^{4d} \otimes (\mathbb{C} + W) + S_-^{4d} \otimes W$.

$\mathrm{Im}\, Q = ?$

- Rotate by $SL(4,\mathbb{C})^R = Spin_6(\mathbb{C})$ will give $\psi \otimes W$
- Rotate by $SO(4)$, get $S_+^{4d} \otimes \mathbb{C}$

$$\mathrm{Ker}\, Q / \mathrm{Im}(\text{rotation}) = (S^+/\psi) \otimes W + S^- \otimes W^*.$$

$$3 \quad + \quad 6$$

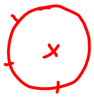
$SU(2) \times SL(3)$ ⟶ $3 \wr z_i$, $6 \wr \partial_{w_j} \wedge \partial_{z_k}$

twist ⇒ $\cap$|

$SO(4) \times Spin(6)$

- Holom. twist of theory on D3 brane ≡ hol CS on $\mathbb{C}^{2|3}$

SUSY are ∂_{ε_i} and $\varepsilon_i \partial_{w_j}$.

A closed string state give a single trace deformation of the theory on the brane  (~bulk deformation).

- Deformat$^{\underline{n}}$ theory on the brane ($\sim$ dga $\Omega^{0,*}(\mathbb{C}^2)[\varepsilon_i]$)
$= HH^*(\Omega^{0,*}(\mathbb{C}^2)[\varepsilon_i]) \simeq HH^*(\mathcal{O}_{\mathbb{C}^2}) \otimes HH^*(\Lambda^*\mathbb{C}^3)$

$HH^*(\mathcal{O}_{\mathbb{C}^2}) = \mathbb{C}[\omega_i, \partial_{\omega_i}]$ (~poly. v.f./$\mathbb{C}^2$; ∂_{ω_i} odd), $\quad HH^*(\Lambda^*\mathbb{C}^3) = \mathbb{C}[\varepsilon_i, \partial_{\varepsilon_i}]$ (∂_{ε_i} even)

$HH^*(\mathbb{C}[\varepsilon_i]) \simeq HH^*(\mathbb{C}[z_i])$ as $\mathbb{C}[z_i], \mathbb{C}[\varepsilon_i]$ are Koszul dual alg.

So $\mathbb{C}[\varepsilon_i, \partial_{\varepsilon_i}] \simeq \mathbb{C}[z_i, \partial_{z_i}]$ w/ $\underline{\partial_{z_i} \leftrightarrow \varepsilon_i \ \&\ \partial_{\varepsilon_i} \leftrightarrow z_i}$ Fourier transform

- $\underset{w_i}{\mathbb{C}^2} \subseteq \underset{(w_i, z_j)}{\mathbb{C}^5}$ IIB SUSY are $\partial w_i \wedge \partial z_j$, z_j

 Apply $z_j \longrightarrow \partial_{\varepsilon_j}$, $\partial_{z_j} \longrightarrow \varepsilon_j$

 $\Longrightarrow$ $\partial w_i \wedge \partial z_j \longrightarrow \varepsilon_j \partial w_i$ are $z_j \longrightarrow \partial_{\varepsilon_j}$.

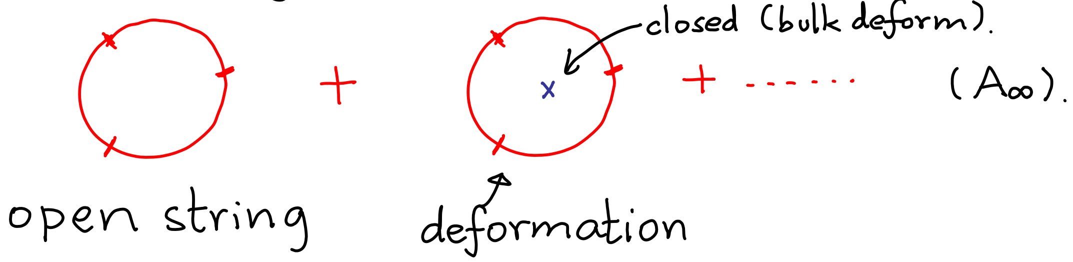
 These are precisely the symmetries of $\mathbb{C}^{2|3}$ which comes from SUSY of $\mathcal{N}=4$ YM.

Eg. In $\mathcal{N}=4$ YM 4d, we have Kapustin-Witten twist comes from family of supercharge $\lambda \partial_{\varepsilon_3} + \mu(\varepsilon_1 \partial_{z_1} + \varepsilon_2 \partial_{z_2})$.

($(\lambda, \mu) \in \mathfrak{t}_{su(3)} \rightsquigarrow [\lambda, \mu] \in \mathbb{P}^1$)

In terms of type IIB on $\mathbb{C}^2_{w_i} \times \mathbb{C}^3_{z_j}$, it is
$\lambda z_3 + \mu(\partial_{z_1} \partial_{w_1} + \partial_{z_2} \partial_{w_2})$.

- Closed string state deforms open strings



Recall: IIB $\rightsquigarrow$ B-model top. string on $\mathbb{C}^5$.

D-branes $\to$ B-branes

IIA $\rightsquigarrow$ $\mathbb{R}^2_A \times \mathbb{C}^4_B$

D-branes $\to$ A/B branes on $\mathbb{R} \times \mathbb{C}^k$ $\quad k \leq 4$

T-duality: IIA on $(\mathbb{R} \times S^1) \times \mathbb{C}^4$

IIB on $\mathbb{C}^\times \times \mathbb{C}^4$

From T-duality, $\mathbb{R} \times S^1 \longleftrightarrow \mathbb{C}^\times$ in top. string.

§ Remaining SUSYs in physical string.

1°. IIA $\rightsquigarrow$ $\mathbb{R}^2_A \times \mathbb{C}^4_B$

D-branes $\to$ A/B branes on $\mathbb{R} \times \mathbb{C}^k$ $\quad k \leq 4$

IIA, $\exists$ 10 remaining $\partial_{z_i} \wedge \partial_{z_j}$, z_k.

2°. IIB $\rightsquigarrow$ B-model top. string on $\mathbb{C}^5$.

D-branes $\to$ B-branes

$\exists$ 15 remaining SUSY, deformed B-model on $\mathbb{C}^5$ by

$\partial_{z_i} \wedge \partial_{z_j}$ (make non-commutative)

z_k (linear superpotential).

<u>Claim</u>: turn on $\partial_{z_1} \wedge \partial_{z_2} \Rightarrow \mathbb{C}^2_{z_1, z_2}$ becomes A-model (w/ large B-field)

Example. Brane on z_2, z_3, z_4, z_5

$\{\text{Fields}\} = \Omega^{0,*}(\mathbb{C}^4)[\varepsilon] \otimes \mathfrak{gl}_N[1]$

$\partial_{z_1} \wedge \partial_{z_2} \rightsquigarrow \varepsilon\, \partial_{z_2}$, deforms the differential.

($\parallel$ brane $\Rightarrow$ same ; $\perp$ brane $\Rightarrow$ ε ($\because$ Fourier)).

$\{\text{fields}\} \xrightarrow{(\partial_{z_1} \wedge \partial_{z_2}\text{ on})} \Omega^*(\mathbb{R}^2) \otimes \Omega^{0,*}(\mathbb{C}^3) \otimes \mathfrak{gl}_N[1]$, via $\varepsilon = dz_2$.

Same as if we treat $\mathbb{C}^2$ as A-model.

A further twist of IIB is top. string on $\mathbb{R}^4_A \times \mathbb{C}^3_B$.

Twist even more, get $\mathbb{R}^8_A \times \mathbb{C}_B$.

IIA Various twists are

$\mathbb{R}^2_A \times \mathbb{C}^4_B$; $\mathbb{R}^6_A \times \mathbb{C}^2_B$; $\mathbb{R}^{10}_A$ ← corresponds to a particular SU(5)-inv. top. twist.

$$\left\{\begin{matrix}\text{Closed string fields} \\ \text{in IIB}\end{matrix}\right\} = \operatorname{Ker}\partial \cap \underbrace{\Omega^{0,*}(\mathbb{C}^5, \wedge^\bullet T\mathbb{C}^5)}_{C^\infty(\mathbb{C}^5)[d\bar z_j, \delta_j],\ \delta_j = \frac{d}{dz_j}}.$$

turn on a background closed string field α $\Rightarrow$ $\bar\partial \mapsto \bar\partial + \{\alpha, -\}$ ← Schouten bracket

Take $\alpha = \partial_{z_1} \wedge \partial_{z_2}$

$\left(\begin{array}{l} \delta_1 = dz_2,\quad \delta_2 = -dz_1 \qquad \{\delta_j, f\} = \frac{df}{dz_j},\ \{\delta_j, z_i\} = \delta_{ij} \\ \partial_{z_1} \wedge \partial_{z_2} = \delta_1 \wedge \delta_2 \qquad \{\delta_1 \wedge \delta_2, -\} = \delta_1 \frac{d}{dz_2} - \delta_2 \frac{d}{dz_1} \\ \quad \delta_1 = dz_2,\ \delta_2 = -dz_1 \qquad = \text{holo. piece of deRham on } \mathbb{C}^2 \end{array}\right)$

$$\Rightarrow \left\{\begin{matrix}\text{Closed string fields} \\ \text{in IIB, w/ } \alpha\end{matrix}\right\} = \Omega^*(\mathbb{R}^4) \otimes \Omega^{0,*}(\mathbb{C}^3, \wedge^\bullet T\mathbb{C}^3)$$

↑ A-model closed string fields.

§ NS5 branes

IIA: $\mathbb{R}^2_A \times \mathbb{C}^4_B \supset pt. \times \mathbb{C}^3$ SUSY <u>NS5 brane</u>

Claim (1). Theory on a <u>single</u> NS5 brane is <u>free</u> limit of string field theory for topo. B-model on $\mathbb{C}^3$

$$\{\text{Fields}\} = \operatorname{Ker} \partial \subseteq PV^{*,*}(\mathbb{C}^3)[2]$$

Fields of ghost # 0 are $\Omega^{0,1}(\mathbb{C}^3, T_{\mathbb{C}^3}) \xrightarrow{\sim} \Omega^{2,1}(\mathbb{C}^3)$,
$\Omega^0(\mathbb{C}^3, \wedge^2\mathbb{C}^3)$, $\Omega^{0,2}(\mathbb{C}^3)$.

This 3-form corresponds to self-dual 3-form on an NS 5-brane.

(2) More generally, $\mathbb{R}^2 \times \underbrace{\mathbb{C}^3_{z_i}}_{NS5} \times \mathbb{C}_w \xrightarrow[\text{linear potential}]{\lambda W} \mathbb{C}$

Then find <u>interacting</u> B-model string on the NS5.
s.t. λ = string coupling constant.

Note: NS5 branes are T-dual to NS5.

IIB $\mathbb{R}^4_A \times \mathbb{C}^3_B \supseteq \mathbb{R}^2 \times \mathbb{C}^2$ NS5-brane.

Theory on a NS5 $\overset{\text{Claim}}{\equiv}$ top. string th. on $\mathbb{R}^2_A \times \mathbb{C}^2_B$

Replace $\mathbb{R}^2$ by $\mathbb{R}^1 \times S^1$ $\rightsquigarrow$ T-dual to IIA picture

- D-brane on $\mathbb{R}^2 \times \underbrace{\mathbb{C}^3_{z_i}}_{NS5} \times \mathbb{C}_w \xrightarrow[\text{linear potential}]{\lambda W} \mathbb{C}$

$$\cup\!\!\mid$$

$$\mathbb{R}_{\geq 0} \times \mathbb{C}^k \times 0$$

$$(*).\ \{\text{Fields}\} = \Omega^*(\mathbb{R}) \otimes \Omega^{0,*}(\mathbb{C}^k)[\underbrace{\varepsilon_1, \dots, \varepsilon_{3-k}}, \delta] \otimes \mathfrak{gl}_N[1]$$

($\varepsilon_1,\dots,\varepsilon_{3-k}$: repr. motion parallel to NS5; δ: motion perpendicular to NS5.)

Boundary condition: fields involving δ are set to 0.

- <u>Boundary fields.</u>

$$\Omega^{0,*}(\mathbb{C}^k)[\varepsilon_1, \dots, \varepsilon_{3-k}] \otimes \mathfrak{gl}_N = \Omega^{0,*}(\mathbb{C}^k, \operatorname{Ext}^{\bullet}_{\mathcal{O}_{\mathbb{C}^3}}(\mathcal{O}_{\mathbb{C}^k}, \mathcal{O}_{\mathbb{C}^k})) \otimes \mathfrak{gl}_N[1]$$

$= \{$fields on a brane in top. B-model on $\mathbb{C}^3$.$\}$

- Turning on superpotential λW,

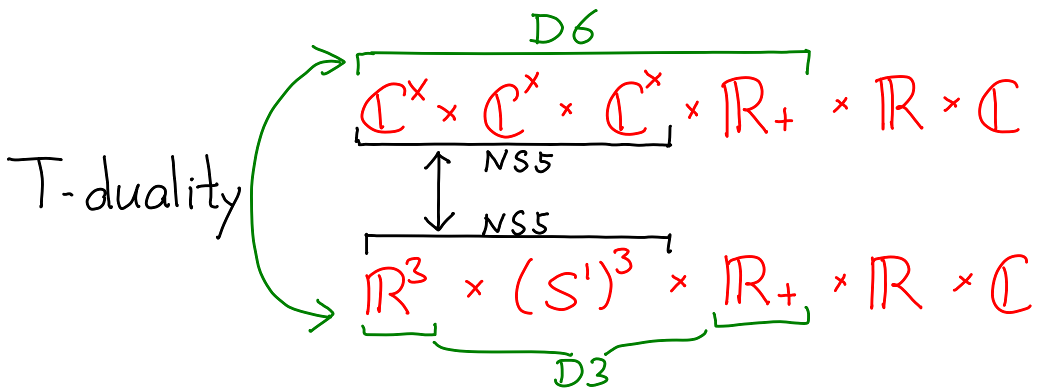
  we get deformation $\lambda \frac{d}{d\delta}$. to (*)

<u>Bulk theory</u>. No operators, (i.e. TFT)

<u>Boundary theory</u> is the theory on a brane in top. string

Eg. $k=3$.

$\Longrightarrow$ TFT on $\mathbb{C}^3 \times \mathbb{R}_+$ (D6 brane) w/ holom CS on the boundary $\mathbb{C}^3 \times 0$ (NS5 brane)



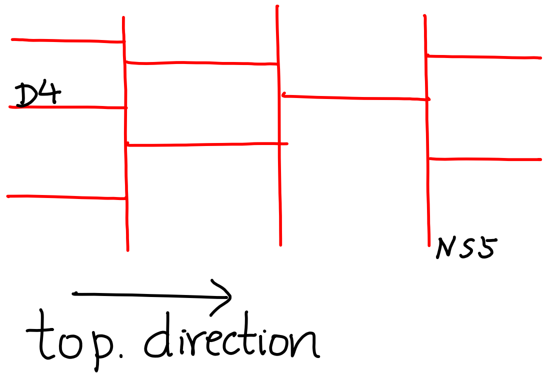
$\Rightarrow$ D3 ending on NS5 gives CS. (as T-dual of hCS).

§ Linear Quiver Gauge theory

4d $\mathcal{N}=2$ quiver gauge theory in the hol. twist.

IIA on $\mathbb{R}^2 \times \mathbb{C}^3 \times \mathbb{C}$

D4-branes on $\mathbb{R} \times \mathbb{C}^2 \times 0$



(linear alg $\rightsquigarrow$ correct bdy. cond.)